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1. What is the range of the real valued function $y = \frac{5}{2x-10}$?
- A) $(-\infty, \infty)$ B) $(-\infty, 0) \cup (0, \infty)$
C) $(-\infty, 5) \cup (5, \infty)$ D) $[0, \infty)$
2. Which of the following is the inverse of the function $f(x) = e^{2x}$?
- A) $g(x) = \ln x$ B) $g(x) = \ln(2x)$
C) $g(x) = \ln \sqrt{x}$ D) $g(x) = e^{-2x}$
3. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ equals the sum of their squares, then
- A) $2ac = ab + b^2$ B) $2ac = a + b$
C) $2ac = a^2 + b^2$ D) $2ac = a^2 - b^2$
4. Which of the following is the equation of the plane containing the lines?
 $\frac{x-1}{-2} = y - 4 = z$ and $\frac{x-2}{-3} = \frac{y-1}{4} = \frac{z-2}{-1}$?
- A) $x + y + z = 0$ B) $x + y + z = 1$
C) $x + y + z = 4$ D) $x + y + z = 5$
5. The parametric equation of the line passing through the point $(1, 2, 3)$ and perpendicular to the xz -plane is given by:
- A) $x = 1, y = 2, z = 3$ B) $x = 1 + t, y = 2, z = 3$
C) $x = 1, y = 2 + t, z = 3$ D) $x = 1, y = 2, z = 3 + t$
6. Which of the following statements is true of the function $f(x) = \frac{x+2}{x^2-3x-10}$?
- A) f is continuous for all real x
B) f has a removable discontinuity at $x = -2$ and a non-removable discontinuity at $x = 5$
C) f has removable discontinuities at $x = -2$ and $x = 5$
D) f has a removable discontinuity at $x = 5$ and a non-removable discontinuity at $x = -2$

7. Consider the functions

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

And

$$g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Which of the following statements about f and g is true?

- A) Both f and g are differentiable at $x = 0$
- B) Both f and g are continuous at $x = 0$ and neither is differentiable at $x = 0$
- C) f is continuous at $x = 0$ but not differentiable at $x = 0$, while g is differentiable at $x = 0$
- D) f is differentiable at $x = 0$, while g is continuous at $x = 0$ but not differentiable at $x = 0$
8. $\int \frac{dt}{\sqrt{x}\sqrt{1+x}}$ equals:
- A) $2\sinh^{-1}(\sqrt{x}) + C$ B) $\sinh^{-1}(\sqrt{x}) + C$
- C) $\ln(\sqrt{x} + \sqrt{1+x}) + C$ D) $\ln(\sqrt{x} - \sqrt{1+x}) + C$
9. What is the volume of the solid generated by rotating the area included between the curve $y = x^2$ and the line $y = x$ about the line $x = 0$
- A) $\frac{2\pi}{15}$ cubic units B) $\frac{\pi}{6}$ cubic units
- C) $\frac{\pi}{3}$ cubic units D) $\frac{\pi}{2}$ cubic units
10. A card is drawn at random from a well-shuffled pack of cards. What is the probability that it is a heart card or a red card or a king?
- A) $\frac{5}{13}$ B) $\frac{6}{13}$ C) $\frac{7}{13}$ D) $\frac{8}{13}$
11. An equilateral triangle is inscribed in a circle of radius 1 centimeter. If a point is taken at random within the circle, what is the probability that the point lies in the triangular region?
- A) $\frac{3\sqrt{3}}{4\pi}$ B) $\frac{3}{4\pi}$ C) $\frac{\sqrt{3}}{4\pi}$ D) $\frac{3}{4}$

12. The series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$
- A) Converges for all p
 B) Diverges for all p
 C) Converges if $p > 1$ and diverges if $0 < p \leq 1$
 D) Converges if $p \geq 1$ and diverges if $0 < p < 1$
13. Which of the following statements is true of the function $f = \frac{1}{x^2}, x \neq 0$?
- A) f is uniformly continuous on $(0, \infty)$
 B) f is not uniformly continuous on $[a, \infty), a > 0$
 C) f is not continuous on $[a, \infty), a > 0$
 D) f is uniformly continuous on $[a, \infty), a > 0$, but not uniformly continuous on $(0, \infty)$
14. Consider the function $f(x, y) = (x^3 + y^3)^{\frac{1}{3}}$. Then
- A) Both $f_x(x, y)$ and $f_y(x, y)$ fail to exist on the line $y = x$
 B) Both $f_x(x, y)$ and $f_y(x, y)$ fail to exist on the line $y = -x$
 C) Both $f_x(x, y)$ and $f_y(x, y)$ fail to exist on the line $y = 0$
 D) Both $f_x(x, y)$ and $f_y(x, y)$ fail to exist on the line $x = 0$
15. If $u = \frac{y-x}{1+xy}$ and $v = \tan^{-1}y - \tan^{-1}x$, then
- A) $u = \tan v$ B) $v = \tan u$ C) $u = \sin v$ D) $u = \cos v$
16. Consider the following three statements about Riemann integrability of a function f on $[a, b]$
- I. If f is monotonic on $[a, b]$, then f is Riemann integrable on $[a, b]$
 II. If f is bounded and continuous on $[a, b]$ except possibly at a or b , then f is Riemann integrable on $[a, b]$
 III. If f is bounded and continuous on $[a, b]$ with only a finite number of points of discontinuities in $[a, b]$, then f is Riemann integrable on $[a, b]$.
- A) Statements I and II are correct B) Statements II and III are correct
 C) All the three Statements are correct D) None of the three statements are correct

17. Let f be an absolutely continuous function on $[a, b]$. Choose the correct statement.
- A) f is of bounded variation on $[a, b]$
 B) f is not of bounded variation on $[a, b]$
 C) f may or may not be bounded variation on $[a, b]$
 D) None of these statements is true
18. What does the equation $|z|^2 = \operatorname{Im}(z)$ represent in the Argand plane?
- A) The imaginary axis
 B) The upper half plane
 C) The circle centred at $\frac{1}{2}i$ and of radius $\frac{1}{2}$
 D) The unit circle centred at the origin
19. Which of the following expressions is equal to $(\sin\theta + i\cos\theta)^4$?
- A) $(\cos\theta + i\sin\theta)^4$ B) $(\cos\theta - i\sin\theta)^4$
 C) $(\sin\theta - i\cos\theta)^4$ D) $(\sin\theta + i\cos\theta)^2$
20. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n!z^n}{n^n}$ is given by
- A) $\frac{1}{e}$ B) e C) 0 D) ∞
21. The largest order of the cyclic group contained in $Z_6 \times Z_8$ is:
- A) 12 B) 18 C) 24 D) 48
22. Let Q be the Quaternion group with centre $Z(Q)$. Then the quotient group $Q/Z(Q)$ is:
- A) a cyclic group of order 4 B) a Klein four-group
 C) a group of order 2 D) a group of order 8
23. Let G be the group of 2×2 non-singular matrices under matrix multiplication. Let H be the subset consisting of lower triangular matrices of the form $\begin{pmatrix} a & 0 \\ c & d \end{pmatrix}$. Then
- A) H is not a proper subgroup of G
 B) H is not a subgroup of G
 C) H is a subgroup, but not a normal subgroup of G
 D) H is a normal subgroup of G

24. Let G be a group of order 15. Then the number of Sylow subgroups of G of order 3 is:
 A) 0 B) 1 C) 2 D) 3
25. Let S_3 be the permutation group on three symbols with identity element e . Then the number of elements of S_3 which satisfy the equation $x^2 = e$ is:
 A) 1 B) 2 C) 3 D) 4
26. Consider the ring $S = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b \in \mathbb{R} \right\}$ with the usual addition and multiplication of matrices and the set $T = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \in \mathbb{R} \right\}$. Choose the correct statement.
 A) T is a subring of S with same identity is that of S
 B) T is a subring of S with an identity different from that of S
 C) T is not a subring of S
 D) None of these statements is true
27. The units of the Euclidean domain $Z[i]$ are:
 A) ± 1 B) $\pm i$ C) $\pm 1, \pm i$ D) None of these
28. Choose the incorrect statement:
 A) $[Q(\sqrt{2}, \sqrt{3}) : Q(\sqrt{2})] = 2$ B) $[Q(\sqrt{2}, \sqrt{3}) : Q(\sqrt{3})] = 3$
 C) $[Q(\sqrt{2}) : Q] = 2$ D) $\sqrt{2}$ and $\sqrt{3}$ are algebraic over Q
29. Squaring the circle is impossible because:
 A) $[Q(\sqrt{\pi}) : Q]$ is a power of 2 B) $[Q(\sqrt{\pi}) : Q]$ is a power of 3
 C) $[Q(\sqrt{\pi}) : Q]$ is finite D) $[Q(\sqrt{\pi}) : Q]$ is not a power of 2
30. If $A = \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}$ and I is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then which of the following is the zero matrix?
 A) $A^2 + A - 5I$ B) $A^2 - A - 5I$
 C) $A^2 + A + 5I$ D) $A^2 - A + 5I$

37. Consider the map $F : R^3 \rightarrow R^2$ defined by $F(x, y, z) = (x + y, y + z)$, Then

- A) F is neither linear nor one to one
- B) F is neither linear nor onto
- C) F is linear and has zero kernel
- D) F is linear and has a nonzero subspace as kernel

38. Which of the following is a solution of the differential equation $x \frac{dy}{dx} - 2y = x^3 e^x$?

- A) $y = x^2$
- B) $y = x^2 (e^x + c)$
- C) $y = \sin x$
- D) $y = \ln x$

39. What is the solution of the differential equation

$$\left(x \sec\left(\frac{y}{x}\right) + y\right) dx - x dy = 0$$

with initial condition $y(1) = 0$?

- A) $x = e^{\sin\left(\frac{y}{x}\right)}$
- B) $x = e^{\cos\left(\frac{y}{x}\right)}$
- C) $x = e^{\sin\left(\frac{x}{y}\right)}$
- D) $x = e^{\cos\left(\frac{x}{y}\right)}$

40. What is the particular integral of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 7 \frac{\partial^2 z}{\partial x \partial y} + 3 \frac{\partial^2 z}{\partial y^2} = e^{x-y}?$$

- A) e^{x-y}
- B) $\frac{1}{12} e^{x-y}$
- C) $\frac{1}{15} e^{x-y}$
- D) $\frac{1}{20} e^{x-y}$

41. Which of the following partial differential equation is hyperbolic?

- A) $\frac{\partial^2 u}{\partial x^2} + 6 \frac{\partial^2 u}{\partial x \partial y} + 3 \frac{\partial^2 u}{\partial y^2} = 0$
- B) $2 \frac{\partial^2 u}{\partial x^2} + 4 \frac{\partial^2 u}{\partial x \partial y} + 3 \frac{\partial^2 u}{\partial y^2} = 0$
- C) $4 \frac{\partial^2 u}{\partial x^2} + 4 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$
- D) $\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$

42. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$ be a topology on X . Then
- A) (X, τ) is a T_1 space
 - B) (X, τ) is a T_2 space
 - C) (X, τ) is neither a T_1 space nor a T_2 space
 - D) (X, τ) is both a T_1 space and a T_2 space
43. Which of the following statements is true?
- A) For $1 \leq p \leq \infty$, the metric space l^p is separable
 - B) For $1 \leq p \leq \infty$, the metric space l^p is complete
 - C) For $1 \leq p \leq \infty$, the closed unit ball in l^p is compact
 - D) For $1 \leq p < r \leq \infty$, the normed space l^r is contained in l^p
44. Choose the correct statement from among the following:
- A) The supremum norm on $C[a, b]$ comes from an inner product
 - B) $C[a, b]$ is not complete with respect to the supremum norm
 - C) $C[a, b]$ is complete with respect to $\|\cdot\|_2$
 - D) $C[a, b]$ is complete with respect to the supremum norm
45. Consider two different inner products in \mathbb{R}^2
 IP 1 defined by $\langle u, v \rangle = u_1v_1 + (u_2v_1 + u_1v_2) + 2u_2v_2$
 for all $u = (u_1, v_1)$ and $v = (u_2, v_2) \in \mathbb{R}^2$ and IP 2 the standard inner product on \mathbb{R}^2 .
 Then the angle between $(1, 0)$ and $(0, 1)$ is:
- A) $\frac{\pi}{4}$ with respect to IP 2
 - B) $\frac{\pi}{4}$ with respect to IP 1
 - C) $\frac{\pi}{2}$ with respect to both IP 1 and IP 2
 - D) $\frac{\pi}{4}$ with respect to both IP 1 and IP 2

46. If $y - 2x + c = 0$ is a tangent to the parabola $y^2 = x$, then the value of c is
 A) $-\frac{1}{8}$ B) -1 C) 2 D) -2
47. If $A = \cos(\cos x) + \sin(\cos x)$, then the least and greatest values of A are
 A) 0 and 2 B) -1 and 1 C) $-\sqrt{2}$ and $\sqrt{2}$ D) 0 and $\sqrt{2}$
48. The values of $\sum_{r=1}^{18} \cos^2(5r)^\circ$, where x° denotes the x degrees, is equal to
 A) 0 B) $7/2$ C) $17/2$ D) $25/2$
49. The value of $\int_c \frac{e^{2z}}{(z-1)^5}$, where c is the circle $|z|=2$ is
 A) $\frac{4\pi}{3} ie^2$ B) $\frac{8\pi}{3} ie^2$ C) $\frac{16}{3} ie^2$ D) $\frac{16}{24} ie^2$
50. Let $f: \mathbb{C} \rightarrow \mathbb{C}$ be defined by $f(z) = \frac{1-e^{-z}}{z}$ for $z \in \mathbb{C}$. For this function, the point $z=0$ is
 A) an essential singularity B) a pole of order zero
 C) a pole of order one D) a removable singularity
51. $\gamma: [0,1] \rightarrow \mathbb{C}$ is defined by $\gamma(t) = 2e^{2\pi it}$. Then $n(\gamma, \frac{1}{2})$ is
 A) Not defined B) 2 C) 1 D) 0
52. The number of fixed points of the Mobius-transformation $S(z) = az + b, a \neq 0, a \neq 1$ are:
 A) 2 B) 1 C) 0 D) 3
53. If $|z-3i| = |z+3i|$, the locus of z is:
 A) real axis B) imaginary axis
 C) circle $x^2 + y^2 = 1$ D) parabola $y^2 = 9x$
54. The number of elements of order 3 in the alternating group A_4 is:
 A) 7 B) 2 C) 8 D) 5

55. How many (non-isomorphic) groups of order 51 are there
 A) 4 B) 3 C) 2 D) 1
56. Find the number of non-zero elements in the field Z_p which are squares *ie.* of the form m^2 , $m \in Z_p$, $m \neq 0$, where p is an odd prime number
 A) $\frac{p-1}{2}$ B) $\frac{p}{3}$ C) $\frac{p+1}{2}$ D) p
57. The number of group homomorphisms from the symmetric group S_3 to $\mathbb{Z}/6\mathbb{Z}$ is
 A) 6 B) 2 C) 3 D) 1
58. Let G be a cyclic group of order 10. For $a \in G$, let $\langle a \rangle$ denote the subgroup generated by a . How many elements are there in the set $\{a \in G \mid \langle a \rangle = G\}$
 A) 3 B) 4 C) 5 D) 1
59. The system $x+y+2z=a_1$, $-2x-z=a_2$, $x+3y+5z=a_3$ has no solution, then
 A) $a_3=a_2$ and $a_1 \neq 0$ B) $a_3=a_2=a_1=0$
 C) $a_3=3a_1$ and $a_2=0$ D) $a_2=-3a_1$ and $a_3=0$
60. The rank of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ by $T(a, b, c) = (a+2b-c, b+c, a+b-2c)$ is
 A) 1 B) 2 C) 3 D) 0
61. Let A be a nilpotent linear transformation on a finite dimensional vector space V over reals. Which of the following is true about A
 A) A is invertible
 B) $I-A$ is invertible
 C) Eigen values of A are of absolute value 1
 D) A has ' n ' distinct eigen values, where n is the dim of V
62. The geometrical effect of the linear transformation associated with the matrix $\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$ is
 A) rotation by an angle $\frac{\pi}{2}$
 B) stretching along Y-axis and a reflection with respect to Y-axis
 C) a stretching along X-axis
 D) reflection with respect to X-axis

63. Let A be a 3×3 matrix with complex entries, whose eigenvalues are $1, \pm 2i$. Suppose that for some $\alpha, \beta, \gamma \in \mathbb{C}$, $\alpha A^{-1} = A^2 + \beta A + \gamma I$, where I is 3×3 identity matrix. Then (α, β, γ) equals
- A) $(-1, -4, 4)$ B) $(-1, 4, -2)$ C) $(-1, -2, 4)$ D) $(4, -1, 4)$
64. Let $M = \begin{pmatrix} 1 & a & b \\ 0 & 2 & c \\ 0 & 0 & 1 \end{pmatrix}$ where $a, b, c \in \mathbb{R}$, then M is diagonalizable if and only if
- A) $a = bc$ B) $b = ac$ C) $c = ab$ D) $a = b = c$
65. The general solution of the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ is
- A) $y(x, t) = \phi(x + ct)$ B) $y(x, t) = \phi(x + ct) + \chi(x - ct)$
 C) $y(x, t) = \phi(x - ct)$ D) No general solution exist
66. For the equation $(x^2 + x - 2)^2 y'' + 3(x + 2)y' + (x - 1)y = 0$, which of the following is correct?
- A) $x = -2$ is regular singular point; $x = 1$ is irregular singular point
 B) $x = -2$ is regular singular point; $x = 1$ is regular singular point
 C) $x = -2$ is irregular singular point; $x = 1$ is irregular singular point
 D) $x = -2$ is irregular singular point; $x = 1$ is regular singular point
67. The partial differential equation $u_{xx} + x^2 u_{yy} = 0$ is of
- A) parabolic B) hyperbolic C) straight linear D) elliptic
68. The solution of $(12x + 5y - 9)dx + (5x + 2y - 4)dy = 0$ is
- A) $6x^2 - 5xy - y^2 + 9x - 4y = c$ B) $3x^2 - 4xy - y^2 + 9x - 3y = c$
 C) $6x^2 + 5xy - y^2 - 9x - 4y = c$ D) $6x^2 + 5xy + y^2 - 9x - 4y = c$
69. Using Picard's method the approximate solution to the initial value problem $y' = 1 + y^2$, $y(0) = 0$ is
- A) $y(x) = \tan x$ B) $y(x) = x - \frac{1}{3}x^3 - \frac{2}{15}x^5 \pm \dots$
 C) $y(x) = x - \frac{1}{3}x^2 - \frac{2}{15}x^4 \pm \dots$ D) $y(x) = x - \frac{2}{3}x^3 - \frac{1}{15}x^5 \pm \dots$
70. The partial differential equation formed from the equation that represents the set of all spheres whose centre lie along the z-axis is given by
- A) $xp - yq = 0$ B) $yp - 2q = 0$ C) $yp - xq = 0$ D) $xp - 2q = 0$

71. Which of the following are solutions to the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 9 \frac{\partial^2 u}{\partial y^2}$
- A) $\cos(3x-y)$ B) x^2+y^2 C) $\sin(3x-3y)$ D) $e^{-3\pi x} \sin \pi y$
72. Which of the following is an example of parabolic type partial differential equation
- A) Wave equation B) Heat equation
C) Laplace equation D) All the above
73. Let X and Y be two topological spaces which are homomorphic. Then which of the following is not true
- A) If X is connected then Y is also connected
B) If X is compact then Y is also compact
C) If X is Hausdroff then Y is also Hausdroff
D) If X is complete then Y is also complete
74. Which of the following is true?
- A) The set of integers Z with usual metric is a complete metric space
B) $[0, 1]$ is nowhere dense in R with usual topology
C) The set Q of rational numbers can be written as $Q = \bigcap_{n \in N} U_n$, where $\{U_n, n \in N\}$ is a sequence of open sets in R with usual topology
D) If d is a bounded metric in X and d' is an unbounded metric on X then d cannot equivalent to d'
75. Let X be a Hausdroff space. Then which of the following is true
- A) A sequence in X may have more than one limit
B) The diagonal $\{(x,x) | x \in X\}$ is not closed
C) If $f: X \rightarrow Y$ is continuous and Y is Hausroff then $\{(x,y) | f(x)=f(y)\}$ is a closed subspace of $X \times X$
D) There exists a metric space which is not Hausdroff
76. Let τ be the topology on R genertaed by $\{[a, a+1] | a \in R\}$, then
- A) τ is the discrete topology B) τ is the trivial topology
C) τ is countable D) Every singleton set is open but not closed

77. Let $X=\mathbb{R}$ with the topology defined by U is open if and only if $X-U$ is finite or X , then the sequence $\{x_n\}$ where $x_n=n$ for all $n\in\mathbb{N}$
- A) Converges and the limit is unique
 B) The limit of x_n cannot be an integer
 C) Converges to 1
 D) It has no limit points
78. If $1 \leq p < q < \infty$ and $0 \neq x \in \mathbb{R}^p$, then which of the following relation is true always
- A) $\|x\|_p > \|x\|_q$ B) $\|x\|_p \geq \|x\|_q$ C) $\|x\|_p < \|x\|_q$ D) $\|x\|_p \leq \|x\|_q$
79. Let $F: X \rightarrow Y$ be a closed, linear map such that $R(F)=Y$ where X and Y are Banach spaces. Which of the following is true
- A) F is continuous and open B) F is continuous but not open
 C) F is open and discontinuous D) F is neither continuous nor open
80. Let X be an inner product space, Y be a subspace of X and $x \in X$. Let y be a best approximation from Y to x . Then $dist(x, Y)$ is
- A) $\langle x, y \rangle^{1/2}$
 B) $\langle x, x - y \rangle^{1/2}$
 C) $\langle x, x + y \rangle^{1/2}$
 D) $\langle x + y, x - y \rangle^{1/2}$
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